progression are $5^2=1^2\times5^2$, $7^2=1^2\times7^2$, and $35^2=5^2\times7^2$; and the progression is $\frac{1}{25}$, $\frac{1}{49}$, $\frac{1}{1225}$.

Put $p=3$ and $q=2$; then the required progression is $(\frac{1}{91})^2$, $(\frac{1}{119})^2$, $(\frac{1}{221})^2$.

II. Solution by O. S. WESTCOTT, A. M., Sc. D., Maywood, Ill.

Let x^2, y^2, z^2 represent the numbers. Then $1/z^2-1/y^2=1/y^2-1/x^2$. And $1/z^2+1/x^2=2/y^2$, or $y^2(x^2+z^2)=2x^2z^2$, or $y^2/x^2z^2=2/(x^2+z^2)$.

Since the first member of this equation is a square, the second must be.

$2/(x^2+z^2)=4[2(x^2+z^2)]$, and we have to make $2(x^2+z^2)$ a square.

Put $x=7$ and $z=1$; then $2(x^2+z^2)=2(49+1)=10^2$. Hence the numbers are 49, $\frac{49}{25}$, 1; the progression being $\frac{1}{49}$, $\frac{25}{49}$, 1.

Or put $x=\frac{1}{7}$ and $z=1$; then $2(x^2+z^2)=2[(\frac{1}{49})+1]=(\frac{10}{7})^2$, and the numbers are $\frac{1}{49}$, $\frac{1}{25}$, 1; the progression being 49, 25, 1.

III. Solution by SYLVESTER ROBINS, North Branch, N. J.

Let a^2, x^2 and b^2 represent three squares whose reciprocals $1/a^2$, $1/x^2$, and $1/b^2$ are in arithmetical progression.

Then $1/a^2+1/b^2=2/x^2$, and $x^2=2a^2b^2/(a^2+b^2)$, a square.

Expand $\sqrt{2}=1$, $\frac{7}{5}$, $\frac{41}{29}$, $\frac{239}{169}$, $\frac{1393}{985}$, $\frac{8119}{5741}$, etc.

Say $a=1$; $b=7$, 41, 239, 1393, 8119, etc.

Then 1^2, $(2\times1^2\times7^2)/(1^2+7^2)$, 7^2 .....1, $\frac{49}{25}$, 49.

1^2, $(2\times1^2\times41^2)/(1^2+41^2)$, 41^2......1, $\frac{1681}{841}$, 1681.

1^2, $(2\times1^2\times239^2)/(1^2+239^2)$, 239^2......1, $\frac{57121}{28561}$, 57121.

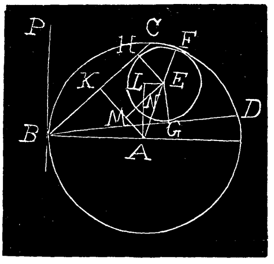
COOPER D. SCHMITT and *G. B. M. ZERR* refer to Problem 78. See solutions of that problem in MONTHLY for March, pages 82—83, by *CHARLES C. CROSS*, *JOSIAH H. DRUMMOND*, *M. A. GRUBER*, and *G. B. M. ZERR*, who also solved the above problem.

AVERAGE AND PROBABILITY.

84. Proposed by L. C. WALKER, Associate Professor of Mathematics, Leland Stanford, Jr., University, Palo Alto, Cal.

From a point in the circumference of a circle two chords are drawn; find (1) the average radius, and (2) the average area of the circle which touches the two chords and the given circle.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.



Let $AB=AF=r$, $HE=GE=FE=x$, $\angle PBD=\theta$, $\angle PBC=\varphi$.

Then $BD=2r\sin\theta$, $BC=2r\sin\varphi$, $AK=r\cos\varphi$, $AN=r\cos\theta$, $BH=BG=BK+KH=BN+NG$.

$$KH=ME=\sqrt{[(r-x)^2-(r\cos\varphi-x)^2]}$$
$$=\sqrt{[r^2\sin^2\varphi-2rx(1-\cos\varphi)]}.$$
$$NG=LE=\sqrt{[(r-x)^2-(r\cos\theta+x)^2]}$$
$$=\sqrt{[r^2\sin^2\theta-2rx(1+\cos\theta)]}.$$
$$\therefore\ r\sin\varphi+\sqrt{[r^2\sin^2\varphi-2rx(1-\cos\varphi)]}$$

$$=r\sin\theta+\sqrt{[r^2\sin^2\theta-2rx(1+\cos\theta)]}.$$

$$\therefore\ x=\frac{2r\,[(\sin\theta-\sin\varphi)\sin(\theta+\varphi)-(\sin\theta-\sin\varphi)^2]}{(\cos\theta+\cos\varphi)^2}$$

$$=2r[\sin\tfrac{1}{2}(\theta+\varphi)\sin\tfrac{1}{2}(\theta-\varphi)\sec^2\tfrac{1}{2}(\theta-\varphi)-\tan^2\tfrac{1}{2}(\theta-\varphi)].$$

$$\pi x^2=4\pi r^2[\sin\tfrac{1}{2}(\theta+\varphi)\sin\tfrac{1}{2}(\theta-\varphi)\sec^2\tfrac{1}{2}(\theta-\varphi)-\tan^2\tfrac{1}{2}(\theta-\varphi)]^2.$$

Let L=average length, $\triangle$=average area.

$$\therefore\ L=\frac{\int_0^\pi\int_0^\theta x\,d\theta d\varphi}{\int_0^\pi\int_0^\theta d\theta d\varphi}=\frac{2}{\pi^2}\int_0^\pi\int_0^\theta x\,d\theta d\varphi$$

$$=\frac{8r}{\pi^2}\int_0^\pi(\theta\cos^2\tfrac{1}{2}\theta-\sin\theta+\sin\theta\log\sec\tfrac{1}{2}\theta)d\theta=\frac{2r}{\pi^2}(\pi^2-8)=.3789r.$$

$$\triangle=\frac{\pi\int_0^\pi\int_0^\theta x^2d\theta d\varphi}{\int_0^\pi\int_0^\theta d\theta d\varphi}=\frac{2}{\pi}\int_0^\pi\int_0^\theta x^2d\theta d\varphi$$

$$=\frac{32r^2}{3\pi}\int_0^\pi(6\theta\cos^4\tfrac{1}{2}\theta-3\theta\cos^2\tfrac{1}{2}\theta-6\sin\tfrac{1}{2}\theta\cos^3\tfrac{1}{2}\theta+2\sin^3\tfrac{1}{2}\theta\cos\tfrac{1}{2}\theta$$

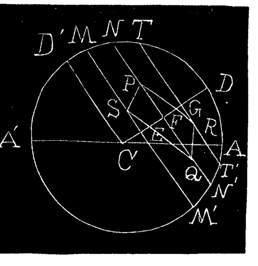
$$-12\sin\tfrac{1}{2}\theta\cos^3\tfrac{1}{2}\theta\log\cos\tfrac{1}{2}\theta)d\theta=\frac{4r^2}{3\pi}(3\pi^2-28)=.2174\pi r^2.$$

85. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Science and Mathematics, Chester High School, Chester, Pa.

Two points are taken at random in a circle and a chord drawn through them; a point is then taken at random in each segment. Find the average area of the quadrilateral formed by joining the four points.

Solution by the PROPOSER.

Let P, Q, R, S be the four random points taken as indicated in the problem, NN' the chord through PQ, MM', TT' the chords through S, R, respectively.



Draw CD perpendicular and CD' parallel to NN'. Let $CD=r$, $CE=u$, $CF=v$, $CG=w$, $NQ=x$, $PQ=y$, $NF=\sqrt{(r^2-v^2)}=z$, $TG=\sqrt{(r^2-w^2)}=t$, $ME=\sqrt{(r^2-u^2)}=s$, $\angle D'CA'=\theta$.

An element of the circle at P is $dvdx$; at Q, $yd\theta dy$; at R, $2tdw$; at S, $2sdu$.